Laser Gyroscopes

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This essay presents a review of the optical gyroscope and associated technologies. Embodied in this review is consideration of the underlying theory responsible for enabling the detection and measurement of rotational motion via non-mechanical devices.

From the initial theory, the essay covers the development of the technology and describes the design considerations that have led to the two current implementations. During this development, several techniques are described on the following pages, which are used to reduce error and improve performance.

The review concludes with examples of applications for laser gyroscopes across several fields of deployment, followed by a discussion of future developments.

Contents

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1. Introduction

Laser gyroscopes offer considerable advantages over their mechanical predecessors. By utilising properties of wave mechanics and light, more durable[1] devices with superior accuracy[1] than the previous mechanical generation may be produced, with no moving parts[1] and no maintenance required[1]. These exhibit a high dynamic range for measurements of rotation about their central axis, often from thousandths of a degree per second*[2] to tens[1] of degrees per second, with typical sample rates near the order 100Hz[1].

2. Ring-laser gyroscopes (RLGs)

One common setup is a ring-laser, and (despite its name) often has a square or triangular cavity. As light propagates in both directions around the "ring", the ring becomes a resonator. Due to the finite speed of light, a continuous rotation of the ring alters the distances that each beam must travel to make a full circuit[1]. In order to resonate, the frequencies of both beams must change to account for the altered circuit lengths[1]. This change in path lengths can be detected by the spatial interference between the beams as they lose phase cohesion, or by the temporal interference due to the mixing of two slightly different frequencies (resulting in low-frequency “beating”)[1]. This change in frequencies can be treated classically, as it is not a result of any relativistic Doppler effect (which cancels out due to the symmetry of the device[3]). Omissions between this simple model and practical implementations cause some minor problems, however these are easily solved without significantly increasing the complexity of the device.

While sources other than lasers have been used[8] in passive gyroscopes, lasers provide narrower bandwidths than their alternatives, resulting in measurements that are more precise. The active gyroscopes, being ring-lasers, would not be possible without laser technology.

Now we will build a mathematical model of this device and derive some important equations that describe its operation.

2.1. Light resonating in a ring

Consider a beam of light, resonating around a closed ring (containing a gain medium) of circumference L, enclosing area A (Fig. A):

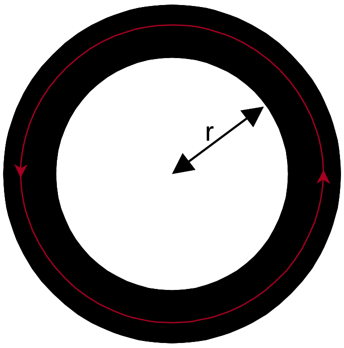

Fig. A Light travelling around a circular path

A quick refresh of geometry reminds us:

$$(111)\quad L = 2\pi r, \quad A = \pi r^2, \quad r = \frac{2A}{L}$$

The time taken for the light to make one pass round the loop, t_{pass}, is given by:

$$(112)\quad t_{pass} = \frac{L}{c},$$

where c is the speed of light in the ring.

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[*] Calculated from data in the referenced material for the typical Nd:YAG *1064 nm* wavelength.

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Now we mark a point along the ring α (which is at a distance r from a rotational axis). If the ring is rotated at a constant angular velocity ω about the centre (Fig. B), the distance that the light must travel to make a full round-trip - starting from α and finishing at α - will be lengthened or shortened by the distance that α travels during that time.

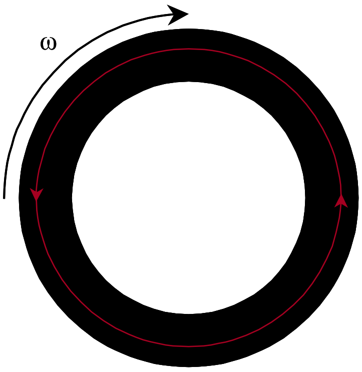

Fig. B Light travelling around a rotating circular path[*]

From geometry, the (tangential) velocity of point α is given by:

$$v_\alpha = r\omega = \frac{2A\omega}{L} \tag{113}$$

With positive ω in the opposite direction to the beam, the decrease in distance for a round trip, δx, is given by:

$$\delta x \simeq v_\alpha t_{pass} = \frac{2A\omega}{c} \tag{114}$$

The corresponding difference between distances travelled for two beams travelling in opposite directions through the ring would therefore be

$$\Delta x = \delta x - (-\delta x) = 2\delta x = \frac{4A\omega}{c} \tag{115}$$

As the ring is a resonator, it must satisfy (periodic) boundary conditions, which requires it to fit an integral number of wavelengths of the beam into one rotation:

$$n\lambda = L \tag{116}$$

Therefore, a shift in resonant frequency must occur under rotation, to satisfy the new path length:

$$\lambda = \frac{c}{f} = \frac{L}{n} \quad \therefore \quad \frac{n}{L} = \frac{1}{\lambda} \tag{117}$$

$$f = c\frac{n}{L} = \frac{c}{\lambda} \tag{118}$$

$$f_\pm = \frac{cn}{L \pm \delta x} = \frac{cn}{L \pm L\dfrac{2A\omega}{Lc}} = \frac{c}{\lambda\left(1 \pm \dfrac{2A\omega}{cL}\right)} \tag{119}$$

2.2. Two counter-propagating beams resonating in a rotating ring

We now introduce a second beam, almost identical to the first, but travelling in the opposite direction around the ring (Fig. C). With no rotation, these will form simple standing waves in the ring, analogous to those formed in a typical laser cavity[†].

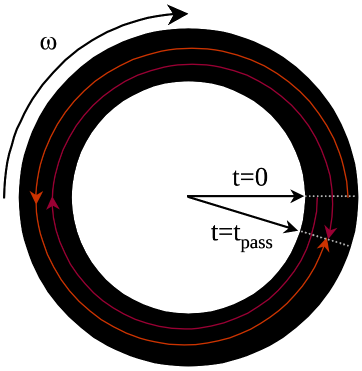

Fig. C Two counter-propagating beams in the ring

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[*] A non-circular path may be approximated as a circular path via vector calculus and geometry. The equations involving area instead of radius are equally valid for non-circular paths[5]

[†] This cyclic cavity presents periodic boundary conditions rather than the $E = 0$ fixed boundary condition of a typical linear cavity. The wave must periodically repeat some pattern an integral number of times for each circuit of the ring.

When the angular velocity of the ring is non-zero, a difference in perceived resonator-length emerges between the two beams, as shown in equation *(115)*. This causes the resonant frequencies for each direction of propagation to diverge according to equation *(119)*.

Where $|2A\omega/cL| \ll 1$, i.e. $|v_\alpha| \ll c$, (the resonator is rotating with non-relativistic speeds), the binomial approximation may be applied to equation *(119)*:

$$(120)\quad f_\pm \simeq \frac{c}{\lambda}(1 \mp \frac{2A\omega}{cL})$$

Therefore, the resonant frequency of each beam changes by:

$$(121)\quad \Delta f = \pm \frac{2A\omega}{\lambda L}$$

As the beams mix at the detector, this results in "beating", where the beat frequency is:

$$(122)\quad f_{beat} = 2\Delta f = \frac{4A\omega}{\lambda L}$$

Therefore, the angular velocity of the system may be calulated from an easily measurable beat frequency, by:

$$(123)\quad \omega = \frac{\lambda L}{4A} f_{beat}$$

For example, a device with a path in the shape of an equilateral triangle with *10* cm sides: $A = 43$ cm^2, $L = 30$ cm, using a HeNe gain medium ($\lambda = 632.8$ nm) will respond to a rotation rate of *0.01* ° s^{-1} with a beat frequency of *16* Hz (beat period of *62* ms).

2.3. Realistic design for a ring-laser gyroscope

In reality, light does not typically travel in circular paths, and a completely closed loop would make measurements somewhat difficult. By splitting the "loop" as shown in Fig. D, the beating may be measured by a detector with a suitably high temporal resolution.

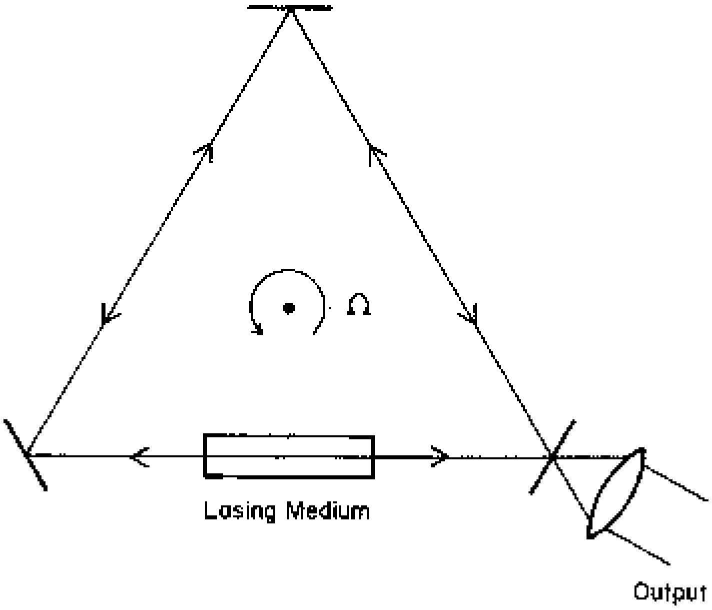


Fig. D   A typical laser gyroscope, with rotation Ω. Adapted from [6]

Here, the two beams are initially in-phase and have the same wavelength. On application of a steady rotation, the resulting difference in path-lengths between the beams results in a phase difference[4], which the lasing medium converts to a frequency difference[5], causing the beating at the detector.

2.4. Enter the third dimension

If the axis of rotation is not perpendicular to the plane of the ring, the measured rate of rotation will be less than the actual rate of rotation, specifically:

For an angle θ between then axis of rotation and the plane of the gyroscope,

(241) $\omega_{measured} = \omega_{actual} \sin\theta$

Where the axis of rotation is unknown, three independent laser-gyroscopes may be mounted at right angles to each other, to provide triaxial rate-of-rotation measurements. An example of such a device is pictured below (Fig. E):

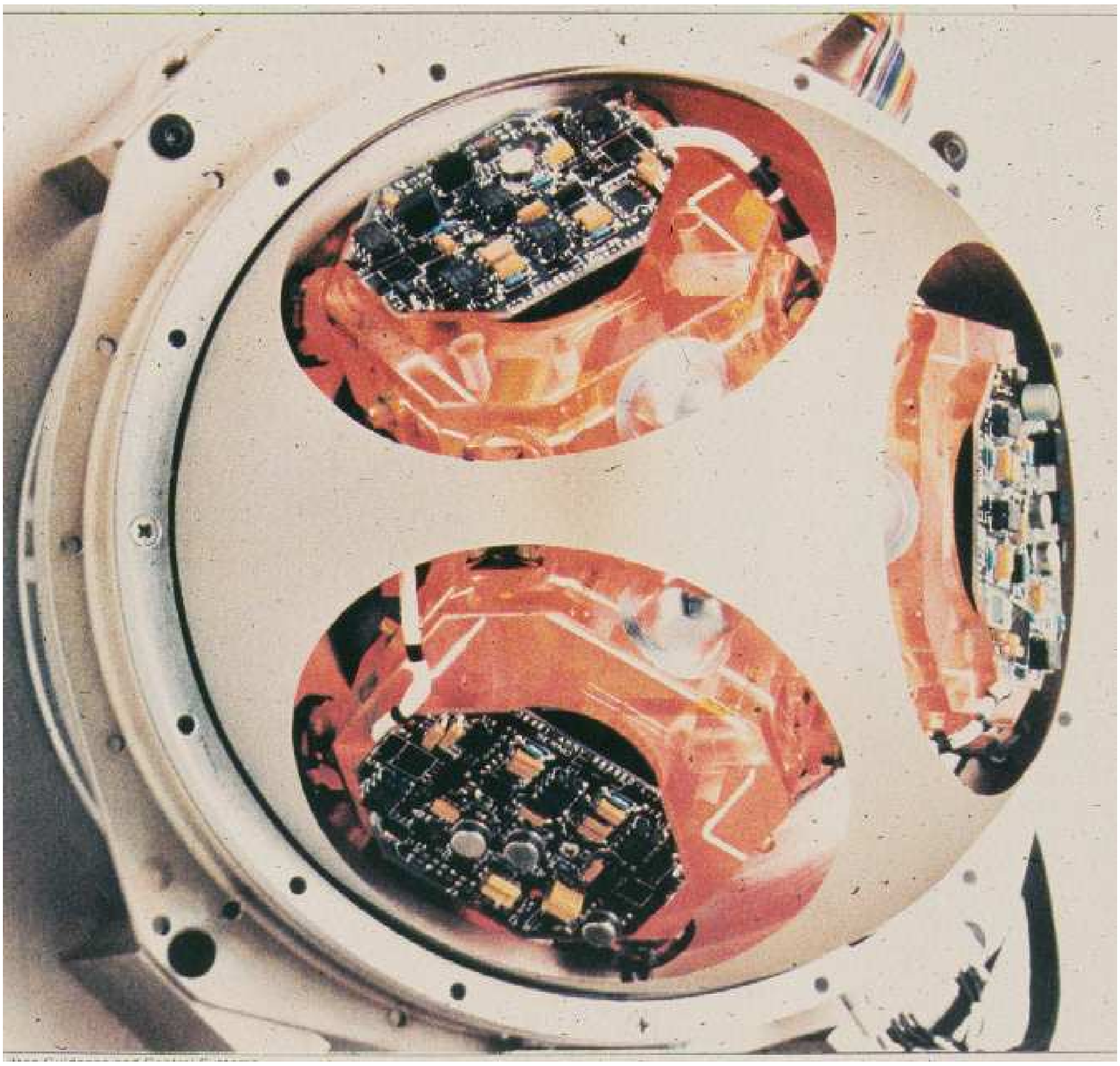
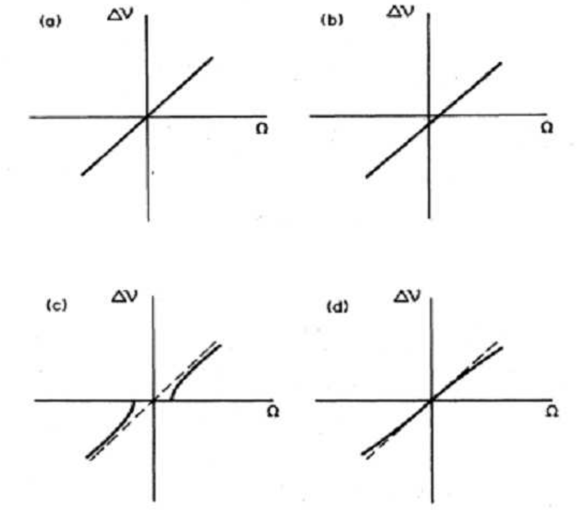


Fig. E "Three axis laser gyro", Prof. A E Siegman, *Stanford University*, retrieved on 10-Dec-2010 from: *http://www.stanford.edu/~siegman/ring_laser_gyros/Three%20axis%20laser%20gyro%20med.jpg*

3. Design Considerations

The model used so far is very simple, and lacks certain important features. The effects of these omissions can cause the device to respond non-linearly, or not to respond at all to some small rotations. Imperfections in the cavity are mostly responsible for the former problems, while the latter are due to interactions between the beams.



Beat note vs input rotation rate in a ring laser gyro. (a) The ideal case, a straight line through the origin; (b) a linear relationship with a nonzero null shift; (c) frequency locking; and (d) nonlinearities in the response (variable scale factor).

Fig. F Different deviations from the ideal response curve for different sources of error
Adapted from [6]

3.1. Lock-in

Slight backwards scattering of the beams results in a small interaction between them in the resonator. When the angular velocity of the system is small, the frequency difference between the beams is also very small, allowing this interaction to cause unintended mode-locking, where both beams will lock to the same frequency, as shown in Fig. G. While this threshold rotation value can be calculated[6], little can be done to prevent the mode-locking from occurring within that threshold.

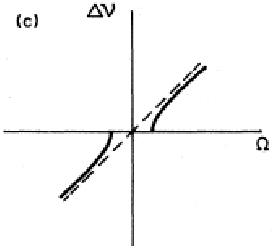


Fig. G The frequency shift (Δυ) vanishes for small rates of rotation (Ω) due to mode-locking.
Acquired from [6]

Application of an electric field across the lasing medium may utilise the Faraday effect to alter the polarisation[6] of the beams in order to reduce their interaction, however this also alters the frequencies of the beams[5], resulting in another source of potential error.

A different approach is to continuously rotate the system by a fixed amount (much larger than the threshold value), in order to keep the device's angular velocity out of mode-locking range. Mechanical rotation introduces a lot of error in the measurements[5], as the rate of mechanical rotation is not known exactly, or perfectly steady*. A more successful approach is "dithering": apply a small oscillation to the angular velocity. There is no net measured effect of this when averaged over several oscillation cycles, as the clockwise rotation cancels out the anticlockwise rotation. This oscillation reduces the amount of time that the angular velocity of the system is within the mode-locking threshold, allowing smaller rates of rotation to be measured. One negative effect of this method is the production of non-linear "dead-bands", where the rotational frequency is an integer multiple of the "dithering" frequency[5].

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[*] Bringing us full-circle back to the original problems with mechanical gyroscopes

Finally, by oscillating the position of one of the cavity mirrors slightly, a Doppler shift can be introduced, increasing the spacing between the frequency of a back-scattered wave and the resonating wave propagating in the same direction[6].

3.2. Null-shift

In a gas laser, atoms in the cavity circulate[7][5], resulting in a slight variation of refractive index depending on the direction of a beam through the medium[5]. This causes a slight frequency-shift between the two beams and a non-zero beat frequency when the system is stationary[7]. This displaces the response curve of the device[1].

One simple solution to this problem is to use two discharge tubes, discharging in opposite directions with respect to the beam path:

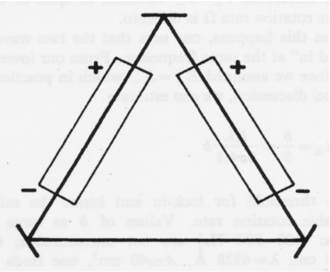

Fig. H Two discharge tubes in the ring laser, with discharges directed in opposite directions in order to cancel out the frequency shift due to flowing of gain atoms.
Acquired from [5]

The frequency shifts from each tube are equal and opposite, cancelling out the null-shift.

3.3. Other error

Even with the previous corrections, noise and non-linear optical behaviour may cause the response of the device to deviate from a straight line. A well-calculated choice of materials and good quality optics can reduce these errors.

4. Fibre-optic gyroscopes (FOGs)

By not having a gain medium in the ring, the change in path-lengths will manifest as phase displacements rather than frequency shifts. From equation *(114)* and the wave equation, the effects of the interference between these two shifted waves at the detector can be calculated:

One dimensional wave equation:

$$u(x,t) \propto \sin(kx - 2\pi f\,t) \tag{401}$$

Following from equation 114, the waves at the detector are characterised by:

$$u_{\pm}(x_{det},t) = sin\left(k(x_{det} \pm \delta x) - 2\pi f\,t\right) \tag{402}$$

where x_{det} is the value of x at the detector when in a stationary state.

The superposition of both waves gives:

$$\begin{aligned} u(x_{det},t) &= 2\; sin(kx_{det} - 2\pi f\,t)\; cos(k\delta x) \\ &= 2u_0(x_{det},t)\cos(k\delta x) \end{aligned} \tag{403}$$

The detected intensity of the interfering beams, relative to the intensity of one beam (I_0):

$$I_{det} = I_0\, 2cos^2(k\delta x) = I_0\, 2\cos^2\left(\frac{2\pi}{\lambda}\frac{2A\omega}{c}\right) = I_0\left(1+\cos\left(\frac{8\pi A\omega}{c\lambda}\right)\right) \tag{404}$$ [5]

Unfortunately, while high-precision detectors capable of detecting small changes in intensity may exist, very long paths may be needed in order to produce detectable changes in the intensity unless a fast rotational rate is expected. For example, Michelson[8] used a device with $A = 0.21$ km^2 to accurately measure the change in interference patterns due to the rotation of the Earth.

4.1. Theory

One simple way to vastly increase the enclosed area, without increasing the volume of the device to impractical values is to stack several loops:

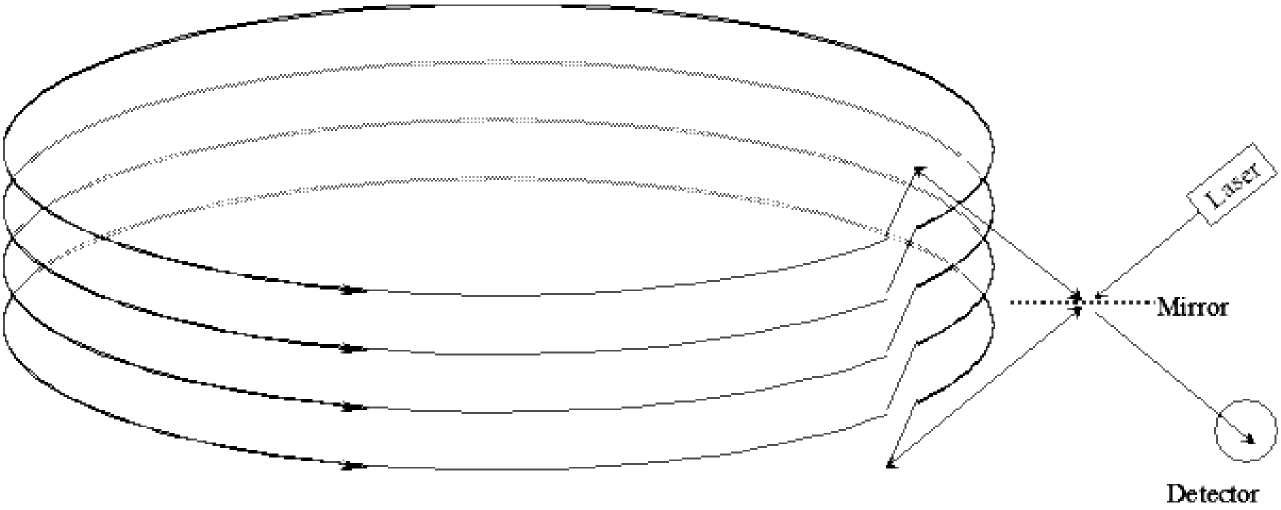

Fig. I Several broken loops, stacked and joined to form a coil

Fibre-optic cables provide a very cheap and convenient way to stack these areas; a coil of N turns, with each single turn enclosing area a provides an effective area A of Na. The reduction in the number of optical components (particularly corner mirrors) combined with the wave-guiding properties of the fibres allows the creation of simple, robust devices requiring no calibration to find their zero-point. As they use small changes in detected amplitude (due to interference), rather than variations in the time-domain (beating) to make measurements they are more susceptible to noise, however they are considerably cheaper to manufacture[9] and suffer from no (or almost no) lock-in compared to ring laser gyroscopes.

As the laser source is external to the ring, solid-state lasers may be used in place of gas-lasers, considerably reducing the power consumption and voltages required for operation.

To provide an area similar to that of Michelson's device (see previous page), using loops enclosing *25* m^2 (approx *5.64* m in diameter), the number of turns needed would be $N = 8{,}400$. As we would be using sensitive photodetectors and lasers with very narrow bandwidth and naturally high spatial coherence (instead of forcing coherence with slits as Michelson did), the number of turns needed could be reduced considerably. Even without this, *8400* turns of a *0.4* mm thick[*] cable results in a coil depth of *3–4* metres; this is far more convenient than Michelson's *0.21* km^2 single-loop[†].

4.2. Error sources in FOGs

Besides the usual sources of error in an optical system[5], some error is due to power transfer between polarisations in the optic fibre, combined with birefringence, which causes a beam to interfere with itself[5]. A highly birefringent fibre may be employed to reduce propagation of the unwanted polarisation. Due to changes in the refractive index as light reaches the end of the fibre, the fibre may act as a weak resonator[5]. This effect can be reduced by gradually changing the refractive index in several steps[5]. The optical Kerr effect (interactions between the electric fields of the beams and the fibre medium) may cause varying phase changes that introduce more error: as with the ring laser gyroscope, a good choice of materials and optical components is essential.

While HeNe lasers typically have very narrow line-widths, diode lasers are being more commonly used in FOGs due to their lower power requirements, smaller size, better durability and reliability.

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[*] Well within the range of typical thicknesses for common fibre optic cables

[†] Although the ~*150* km of fibre optic cable necessary for this many loops would not be so practical!

5. Examples of modern laser gyroscope systems

5.1. Military

Due to the accuracy, robustness and low maintenance needs of optical gyroscope systems, they have been a large success with the military. The notorious *AC-130U* gunship uses ring-laser gyroscopes[10] in addition to standard GPS systems in order to provide fast directional information.

The primary inertial-reference system on the *Boeing 777* uses Honeywell laser gyroscopes[11], as do some of Boeing's other commercial planes including the *757* and the *767*[17].

Another interesting use is on a USAF anti-satellite missile: *ASM-135 ASAT*, which also uses a Honeywell ring laser gyroscope[12].

5.2. Commercial

An example of a discrete (fibre) laser gyroscope device is the *Northrop Grumman LN-251*[13]: a durable six-kilogram device requiring a power of only twenty-five watts. It uses a single solid-state diode laser, and owes its accuracy to fibre-optic gyroscope(s) [13], page 4.

Another notable FOG is the *KVH Industries DSP-1500*[14]: its sensing element is only *1.5* inches (~*38* mm) in diameter[14], coiled to a thickness of approximately *20* mm[14], resulting in a unit weighing just forty grams[14]! Despite this, the device has a very wide range of operating environments[15] and error of less than five degrees per hour[15]. A two-axis variant of this device is also available[14].

Toyota is also developing navigation systems for cars that utilise ring-laser gyroscopes[16].

5.3. Scientific

The Canterbury Ring Laser (C-I)'s successor, C-II, encloses and area of one square metre[18] by a cavity filled with helium and neon (forming a HeNe ring laser)[18]. This was a prototype for a larger[19] project (enclosing *16* m^2) — the "Gross Ring", which measures fluctuations in the rotation rate of the earth[19] to within one part per billion[19].

6. Conclusion

As currently available technology is already capable of providing measurements to higher resolutions than are usually required and the laser gyroscopes are typically combined with other systems[20] (such as GPS/GLONASS), large improvements of the underlying technology are unlikely to be of much importance in future development of it. Instead, improved efficiency and miniaturisation are more likely, as the technology becomes embedded in increasingly mass-produced consumer products such as cars and (size-depending) satellite-navigation devices and mobile phones.

With this in mind, the simpler power requirements, lower cost and lower part-count of FOGs (in comparison to ring-laser devices) are likely to contribute far more to their popularity than their inferior resolution[20] will do to impede it, while ring-laser devices will enjoy increasing use in high-precision situations such as scientific research.

7. Table of figures

8. References

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