6.3. Velocity potential for a dipole

A dipole is formed by placing a source close to a sink of equal strength. When superimposed they cancel out, however when spaced an infinitesimal distance apart, they create an interesting field. This model may be used in many areas, including magnetism and fluid dynamics. .

$$\phi_{pole}\left(A,\vec{x}\right)=\frac{A}{\left|\vec{r}-\vec{x}\right|}\text{; strength } A\text{, centre } \vec{x} \qquad 6.3$$

$$\phi_{source}=\phi_{pole}\left(A>0\right),\ \phi_{sink}=\phi_{pole}\left(A<0\right) \qquad 6.4$$

In this example, the source ϕ_{in} and sink ϕ_{out} both have strength (flow-rate) A. The source is located at the origin, and the sink is displaced a small distance of Δ along the first axis (commonly, the x-axis):

$$\phi=\phi_{in}+\phi_{out}=\phi\left(A,\vec{0}\right)+\phi\left(-A,\vec{0}+\Delta\hat{x}\right) \qquad 6.5$$

$$\phi_{dipole}=\lim_{\Delta\to 0}\phi \qquad 6.6$$

$$=\frac{A}{\left|\vec{r}\right|}-\frac{A}{\left|\vec{r}+\Delta\hat{x}\right|} \qquad 6.7$$

$$=A\left(\frac{1}{\sqrt{\sum x_i^2}}-\frac{1}{\sqrt{\sum x_i^2+2\Delta x_0+\Delta^2}}\right) \qquad 6.8$$

$$\approx A\left(\frac{1}{\left|\vec{r}\right|}-\frac{1}{\left|\vec{r}\right|+\frac{2x_0\Delta+\Delta^2}{2\left|\vec{r}\right|}}\right)\text{, via binomial approximation} \qquad 6.9$$

$$\approx A\left(\frac{1}{\left|\vec{r}\right|}-\frac{2\left|\vec{r}\right|}{2\left|\vec{r}\right|^2+2x_0\Delta}\right)\text{, for } \Delta\ll\left|x_0\right| \qquad 6.10$$

$$\approx\frac{Ax_0\Delta}{\left|\vec{r}\right|^3}\text{, using the same assumption} \qquad 6.11$$

i.e. $$\phi_{dipole}=\frac{\mu x}{\left|\vec{r}\right|^3} \qquad 6.12$$

The potential ϕ is shown in Figure 6-C, while the corresponding gradient field is shown in Figure 6-D.

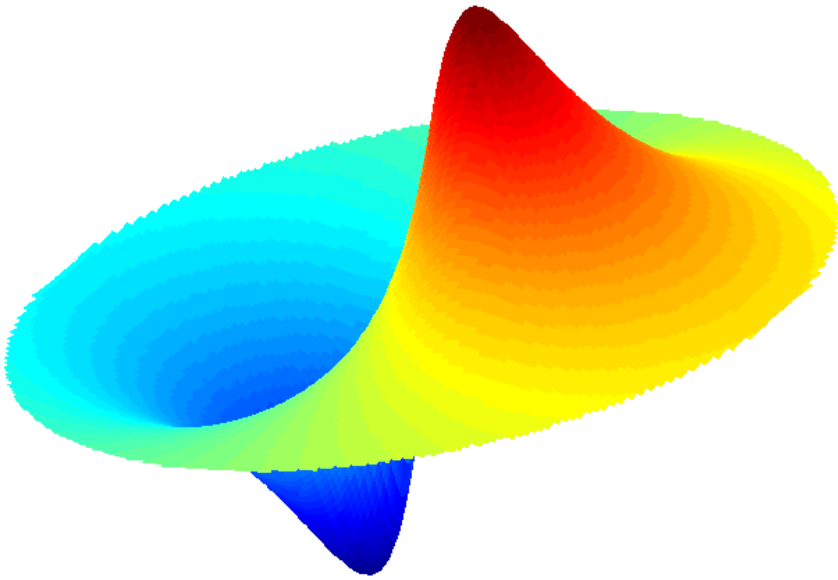

Figure 6-C: Scalar potential for a dipole (e.g. velocity potential).

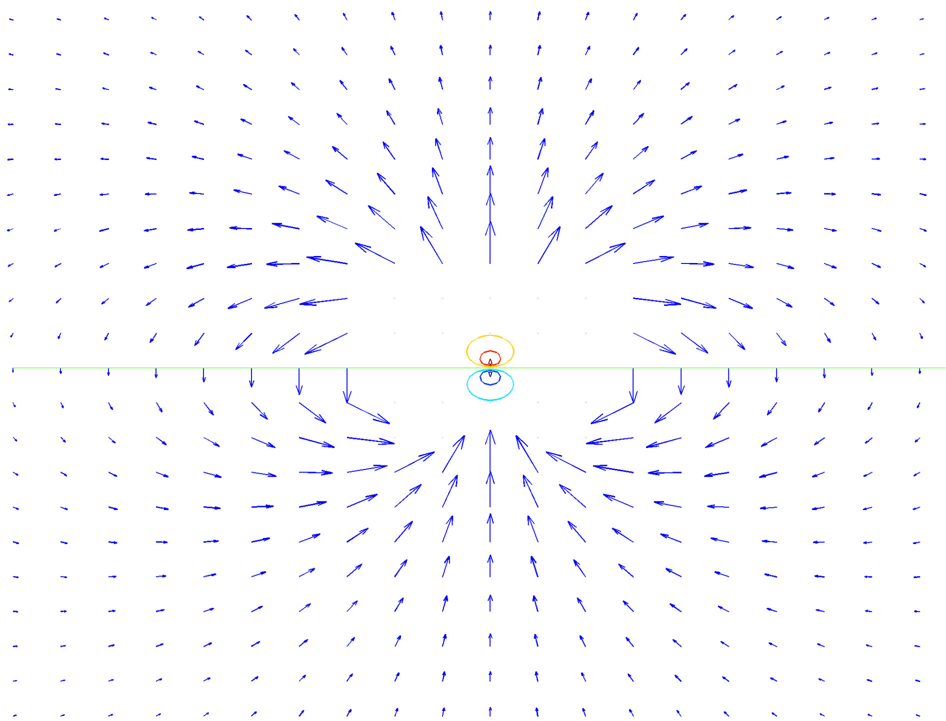

Figure 6-D: Gradient of dipole potential (e.g. fluid velocity field). This is zoomed out in relation to the previous figure; the region illustrated in Figure 6-C is shown in the centre as a contour plot.